FST 6-2 Notes
Topic: Principles of Probability
GOAL
Develop two forms of Addition Counting Principle from basic principles of probability,
and use them to develop theorems for finding probabilities of unions of events (mutually
exclusive or not), and probabilities of complementary events.

SPUR Objectives

A Compute probabilities of events in
various contexts.

D Compute probabilities using the
General and Mutually Exclusive
Forms of the Probability of the Union
of Events and the Probability of
Complements.

Vocabulary

union of sets

disjoint sets

mutually exclusive sets

intersection of sets

complementary events

complement of A, not A

Consider the two experiments.
What would be their possible outcomes?

Experiment 1: A single card is chosen at random from a standard deck of 52 playing
cards. What is the probability of choosing a 5 or a king?
Probabilities: The card chosen can be a 5
The card chosen can be a King

Experiment 2: A single card is chosen at random from a standard deck of 52 playing
cards. What is the probability of choosing a club or a king?
Probabilities: The card chosen can be a club
The card chosen can be a King
The card chosen can be a club + King

What is the difference between the two experiments?
In Experiment 1, the cards can't occur at the same time. In Experiment 2, cards can occur at the same time

Set language	Disjoint	Intersecting
Event language	Mutually Exclusive	Overlapping
Venn diagram	5 King	Club AnB King
	A B	A B
Symbols	$A \cap B = \emptyset$	$A \cap B \neq \emptyset$

Experiment 1   Experiment 2

Mutually Exclusive

Two events are **mutually exclusive** if they cannot occur at the same time (i.e., they have no outcomes in common). $A \cup B$, the **union** of set A and B.

> **Addition Counting Principle (Mutually Exclusive Form)**
>
> If two finite sets A and B are mutually exclusive, then
> $N(A \cup B) = N(A) + N(B)$.

Experiment 1: A single card is chosen at random form a standard deck of 52 playing cards. Suppose you want to choose a 5 or a king. How many outcomes are there?

5: 4 of them
King: 4 of them

$4 + 4 = 8$ outcomes

What is the probability of choosing a 5 or a king?

> **Theorem (Probability of the Union of Mutually Exclusive Events)**
>
> If A and B are mutually exclusive events in the same finite sample space, then $P(A \cup B) = P(A) + P(B)$.

$P(5 \text{ or } king) = P(5) + P(king)$

$\dfrac{4}{52} + \dfrac{4}{52} = \dfrac{8}{52}$

Overlapping Events – Intersection

Occurs when two events are not mutually exclusive; they have outcomes in common. $A \cap B$, the **intersection** of sets A and B.

> **Addition Counting Principle (General Form)**
>
> For any finite sets A and B, $N(A \cup B) = N(A) + N(B) - N(A \cap B)$.

Experiment 2: A single card is chosen at random from a standard deck of 52 playing cards. Suppose you want a club or a king. How many outcomes are there?

Clubs: 13 of them
Kings: 4 of them

$13 + 4 - 1 = 16$ outcomes

What is the probability of choosing a club or king?

> **Theorem (Probability of the Union of Events, General Form)**
>
> If A and B are any events in the same finite sample space, then $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$P(club \text{ or } king) = P(club) + P(king) - P(Both)$

$\dfrac{13}{52} + \dfrac{4}{52} - \dfrac{1}{52} = \dfrac{16}{52}$

Example 1: A pair of six-sided dice is thrown. If the dice are fair, what is the probability that the dice show a sum of 7 or 11?

"or" → +

Sum of 7: $\{(3,4)\ (4,3)\ (5,2)\ (2,5)\ (1,6)\ (6,1)\}$

Sum of 11: $\{(5,6)\ (6,5)\}$

$P(7\ or\ 11) = P(7) + P(11)$

Mutually Exclusive – No overlap

$$\frac{6}{36} + \frac{2}{36} = \boxed{\frac{8}{36}}$$

Example 2: What is the probability that the dice show doubles or a sum of 8?

Doubles: $\{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\}$

Sum of 8: $\{(4,4)\ (6,2)\ (2,6)\ (5,3)\ (3,5)\}$

Not Mutually Exclusive – overlap

$P(Doubles\ of\ 8) = P(Doubles) + P(8) - P(Both)$

$$\frac{6}{36} + \frac{5}{36} - \frac{1}{36} = \boxed{\frac{10}{36}}$$

Complementary Events

Events that are mutually exclusive and their union is the entire sample space

Experiment	Sample Space	Event	Complement
tossing a coin	{heads, tails}	{tails}	{heads}
tossing two coins	{HH, HT, TH, TT}	getting no heads {TT}	getting 1 or 2 heads {HH, HT, TH}
picking an integer from 1 to 100	{$n \in \mathbb{Z} : 1 \leq n \leq 100$}	picking a prime number	picking 1 or a composite number

Not A
Not tails
Not getting no heads
Not picking prime #
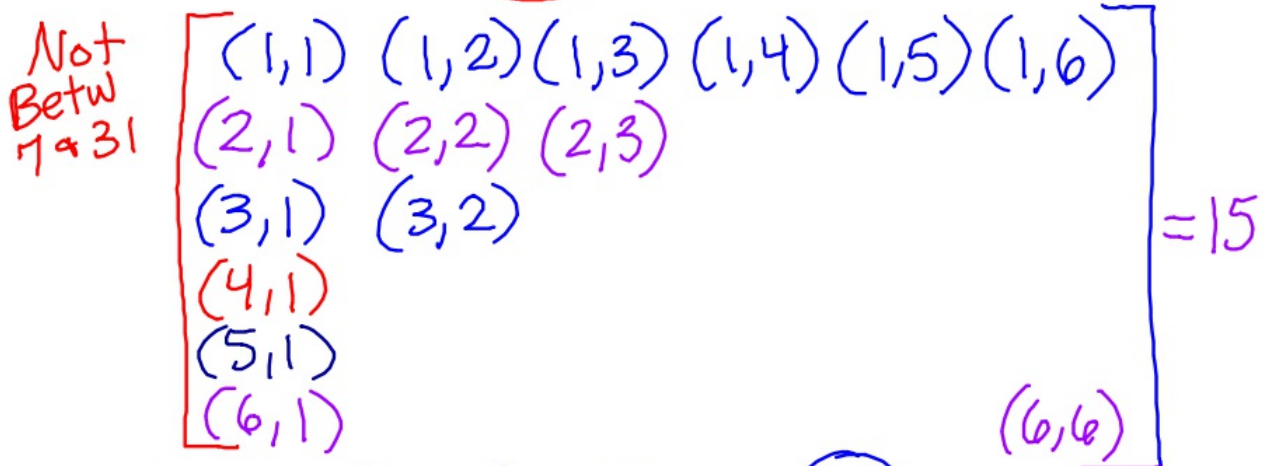
The **complement of an event A** is called **not A**.

Theorem (Probability of Complements)

If A is any event, then $P(not\ A) = 1 - P(A)$.

$P(A) + P(not\ A) = 1$

Probability of Mutually Exclusive events Add to 1

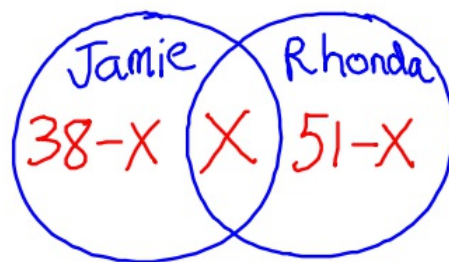$P(Jeans) = \frac{20}{29}$

$P(not\ Jeans) = 1 - \frac{20}{29}$

$$\frac{29}{29} - \frac{20}{29}$$

$$= \boxed{\frac{9}{29}}$$

Example 3: A pair of six-sided dice is thrown. If the dice are fair, what is the probability that the dice show a (product) between 7 and 31?

Not
Betw
7 & 31

$$
\begin{bmatrix}
(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\
(2,1) & (2,2) & (2,3) & & & \\
(3,1) & (3,2) & & & & \\
(4,1) & & & & & \\
(5,1) & & & & & \\
(6,1) & & & & & (6,6)
\end{bmatrix} = 15
$$

$$
P\left(\underset{7 \to 31}{Between}\right) = 1 - \frac{15}{36} = \boxed{\frac{21}{36}}
$$

Example 4: Jamie and Rhonda had to take a make up test after school. Their teacher told them they could come in no earlier than 3:00 and leave no later than 4:00. Jamie took 38 minutes on the test and Rhonda 51 minutes. For at least how many minutes were Jamie and Rhonda taking the test at the same time?

Jamie   Rhonda
38-X (X) 51-X   X = Overlap

$$
60 = 38 - X + X + 51 - X
$$
$$
60 = 89 - 1X
$$
$$
\underline{-89 \quad\quad -89}
$$
$$
\frac{-29}{-1} = \frac{-1X}{-1}
$$

$$
\boxed{X = 29 \text{ minutes overlap}}
$$